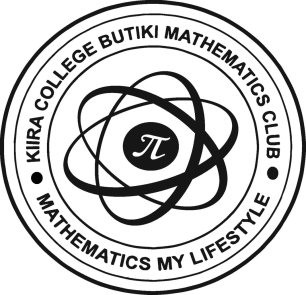
***KIIRA COLLEGE BUTIKI MATHEMATICS CLUB  
  
MATHLETICS CONTEST 2019  
  
JUNIOR MATHLETES CATEGORY SUGGESTED SOLUTIONS***

***Section A:   
Qn.1.*** translates to or 27 .

***Qn.2.*** . Notice that at this point the equation cannot be solved any further; hence is a solution.

***Qn.3.*** Observe that . Note also that when 2401 is multiplied any number of times; the last digit of the product is 1 i.e. for any whole number , will always have 1 as the last digit.

***Qn.4.*** Considering the sketch below; having a circle with center , let’s  
 set the intersection of and to be ; so that   
, . Using the *Pythagorean Theorem* on triangle   
, we have: . This solves   
to implying that . Applying the   
Pythagorean Theorem on triangle , we have that   
 .

***Qn.5.  
Solution 1:*** Let’s consider such a 2-digit number . Since there are 5 odd digits (1, 3, 5, 7 and 9), has got 5 choices as well as ; hence the required number of these 2-digit numbers is .  
***Solution 2:*** Since both digits have to be   
  
odd, these numbers then only appear among the ‘teens’, thirties, fifties, seventies and nineties. Interestingly, in each of these groups, there are exactly five such numbers e.g. (11, 13, 15, 17, 19); (31, 33, 35, 37, 39); (51, 53, 55, 57, 59) etc; hence, a pattern has appeared.

|  |  |
| --- | --- |
| ***Group*** | ***No.*** |
| *Teens* | *5* |
| *Thirties* | *5* |
| *Fifties* | *5* |
| *Seventies* | *5* |
| *Nineties* | *5* |
| ***Total*** | *25* |

***Solution 3:*** Listing these numbers happens to be the most accurate method to obtain the required solution. There are 90 2-digit numbers ; 45 of these are odd and it turns out that 25 of these have both digits as odd numbers. We can trust the accuracy of this method because the time allowed at the ***Mathletics Exercise*** is just enough for short listings like this one.

***Qn.6.*** From the statement of the problem, the region is congruent   
to the three regions (, and ) in its vicinity. If their area   
altogether is , then each of them has an area of . Notice that   
there are exactly 12 such regions; with the same area, outside   
the circles and their common region , but within the   
square. Hence, the required area is .

***Qn.7.*** The required sum is .

***Qn.8.*** Note that ; but ; hence .

***Qn.9.*** From ; . Since and , then .

***Qn.10.*** Let . Increasing by 300% we have . Reducing by 75%, we have . Hence, after the ‘big’ increase and relatively ‘smaller’ decrease, the salary remained the same; which explains Ronnie’s unhappiness.

***Section B:  
Qn.11.*** The counting proceeds as follows: Abel-208; Bob-207; Clarence-206; Dee-205: Abel-204; Bob-203; Clarence-202; Dee-201: Abel-200; Bob-199; Clarence-198; Dee-197: Abel-196 etc. Note that Abel always counts multiples of 4 hence, he will count 4 as well, then Bob will count 3, Clarence will count 2 and Dee will go first in the game; as he counts 1.

***Qn.12.*** Let their ages be two-digit numbers and . Expanding them using place values (or values); they become and respectively. This implies that their sum is ; and their difference is. From the statement of the problem, . Hence, is 5, is 4 and the required ages are 54 and 45 .

***Qn.13.*** Since the number of families with one bicycle is equal to the number of families with three bicycles; let’s assume that each of the families with three bicycles donates one bicycle to those families with only one bicycle; such that we create an apparent situation where all the 35 families have exactly 2 bicycles. Hence, there are bicycles in the village.

***Qn.14.*** We can get the *unique sum* of the square by adding the given major diagonal i.e. . We then complete; the upper row by ; the other major diagonal by ; then the lower row by ; the right column by and lastly the left column by to give the square below.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

***Qn.15.*** Let’s consider trying out each of these suspects. Note that Cecilia and Denise have not been mentioned anywhere; so we can ‘spew them out of the game’. If Bernard stole the phone; Cecilia, Albert and Denise would all be telling the truth, which is not true. If Albert stole the phone; only Bernard would be telling the truth, which is true. Beyond reasonable doubt, Albert is the thief.

***Bonus:*** In the subtraction given below, and represent some two-digit positive integers:

What is the value of ? Could you find the number of pairs of positive integers satisfying the equation?